5 / C: The Taxi Problem

Contents

1 Instructions

Do the preparations first. Then,

Group If you are on-campus, or can otherwise form a group of 3–4, do Variant A.

Solo If you are on your own, read the context in Variant A, then do the guiding questions as outlined in Variant B.

When you are finished, do the Specific Energy/ Energy Density section.

2 Preparations

1. **Stoichiometric calculations**

How would you do each of the following?

(a) Calculate the molar mass of a compound (*e.g.*, $C_2H_{6(g)}$)

SUM UP ATOMIC MASSES OF ATOMS IN MOLECULE.
e.g., $A_C + A_C + 6\,(A_H)$
$= (12.0 + 12.0 + 6.0)\ g/mol = 30.0\ g/mol$

(b) Calculate the number of moles in a compound, given the mass (*e.g.*, 60 g of $C_2H_{6(g)}$)

① FIND MOLAR MASS M
② FIND n BY $n = m/M$

EXAMPLE
$M = 30.0\ g/mol$
$n = m/M = 60\ g \div 30\ g/mol$
$= 60\,g \times \frac{1}{30}\ mol/g$
$= 2.0\ mol$

(c) Calculate the mass of a certain volume of liquid (*e.g.*, 250 cm^3 of ethanol)

MULTIPLY BY ITS DENSITY ρ
e.g., $m_{EtOH} = V \times \rho_{EtOH}$
$= 250\ cm^3 \times 0.789\ \frac{g}{cm^3} = 197\ g$

WRITING OUT UNITS IN INTERMEDIATE STEPS, AND MAKING SURE THEY CANCEL OUT TO SENSIBLE OUTCOMES.

THIS IS AN ESSENTIAL TECHNIQUE FOR REASONING IN THIS & OTHER NUMERICAL PROBLEMS.
OPT C HAS NUMEROUS POSSIBILITIES OF WORKING PROBLEMS ON SOCIETAL SCALE (GJ, TW...) AND YOU SHOULD NOT EXPECT TO SOLVE THEM CORRECTLY UNLESS YOU DO DIMENSIONAL ANALYSES AS A HABIT.

GIGAJOULE

TERAWATTS

2. **Enthalpy calculations**

How would you do each of the following?

(a) Find the ΔH_c° of a "mainstream" compound (*e.g.*, ethanol).

LOOK IT UP IN THE DATA BOOKLET.

e.g., $\Delta H^{\circ}_{c\,EtOH} = 1367 \frac{kJ}{mol}$

(b) Find the q from combustion of a certain number of moles of a compound (*e.g.*, 5.0 mol of ethanol)

$q = \Delta H^{\circ} \cdot n$

e.g., $q = 1367 \frac{kJ}{mol} \times 5.0\ mol$

$= 6835\ kJ$

(c) Find the ΔH_c° of a compound that is not in the Data Booklet (*e.g.*, naphthalene).

LOOK IT UP ON GOOGLE, OR CRC HANDBOOK OF CHEM & PHYS.

3 Variant A: Group

The year is 2000. You have been appointed as the Transport Secretaries of Hong Kong.

Your first task is to decide upon a suitable subsidy to help taxi owners convert their existing gasoline-consuming fleets to LPG-consuming ones.

It is believed that this change will be both **green** (environmentally friendly) and **economically viable** (LPG is cheaper as a fuel). However, changing out the engine is quite expensive and taxi owners request aid from the government to help them do so.

Some data your minions have collected for you:

Cost of gasoline HK$ 1.8 / dm^3

Cost of LPG HK$ 0.6 / dm^3

Fuel consumption of gasoline 13 dm^3 / 100 km

Fuel consumption of LPG Unknown

Cost of engine replacement HK$ 16000

Taxis Usually rented out and driven for two shifts of 10–12 h each.

Because your minions did not study chemistry, they are unable to help you locate other needed information. Use the internet to help you.

(If your group is absolutely stuck, use questions from Variant B to help you move forward.)

4 Variant B: Solo

Read the context in Variant A if you have not already done so.

3. (a) Identify the components in gasoline and LPG respectively.

GASOLINE (US, CANADA) / PETROL (UK) : A MIXTURE OF HYDROCARBONS, USUALLY AROUND C8

LIQUIFIED PETROLEUM GAS : A MIXTURE OF PROPANE C_3H_8 & BUTANE C_4H_{10}

(b) For gasoline, assume it is pure octane.

i. Calculate the mass of 1.00 dm^3 gasoline.

i. 702.5 g

$$m = V \cdot \rho$$
$$= 1.00\ dm^3 \times 0.7025\ kg/dm^3$$
$$= 0.7025\ kg$$
$$= 702.5\ g$$

ii. Calculate the number of moles of octane present in 1.00 dm^3 gasoline.

ii. 6.150 mol

$$n = \frac{m}{M} = \frac{702.5\ g}{114.23\ g/mol}$$
$$= 6.150\ mol$$

YOU MAY HAVE USED DIFFERENT NUMBERS OR ASSUMPTIONS, AND GET TO ANSWERS THAT ARE SLIGHTLY DIFFERENT.

TO CHECK YOUR ANSWERS YOU CAN USE THE SPREADSHEET FOUND IN YOUR GOOGLE DRIVE. SEARCH FOR "5/C TAXI PROBLEM".

iii. Calculate the amount of energy when 1.00 dm^3 of gasoline is completely combusted.

iii. -3.401×10^4 kJ

$\Delta H^\ominus_{c\ OCTANE} = -5530 \frac{kJ}{mol}$

$q = n \cdot \Delta H^\ominus_c$

$= 6.150\ mol \times -5530 \frac{kJ}{mol} = -3.401 \times 10^4$ kJ

iv. Calculate the fuel consumption of gasoline, giving your answer in mol/km.

iv. $0.7995 \frac{mol}{km}$

FUEL CONSUMPTION $= \frac{13\ dm^3}{100\ km} = 0.13 \frac{dm^3}{km}$

FUEL CONSUMPTION in mol $= n_{IN\ 1.0\ dm^3} \times 0.13 \frac{dm^3}{km}$

$= 6.150 \frac{mol}{dm^3} \times 0.13 \frac{dm^3}{km} = 0.7995 \frac{mol}{km}$

v. Calculate the fuel cost of gasoline, giving your answer in HK$/km.

v. $0.234 \frac{HK\$}{km}$

FUEL COST $= 1.8 \frac{HK\$}{dm^3}$

FUEL CONSUMPTION $= 0.13 \frac{dm^3}{km}$

FUEL COST PER km $= 1.8 \frac{HK\$}{dm^3} \times 0.13 \frac{dm^3}{km} = 0.234 \frac{HK\$}{km}$

(c) Assuming that a car would take the same amount of energy to run, regardless of the fuel, calculate the fuel cost of LPG. Assume LPG is all propane.

(c) $0.090 \frac{HK\$}{km}$

ENERGY REQ'd TO DRIVE 1 km $= n_{OCTANE\ FOR\ 1km} \times \Delta H^{\ominus}_{c,\ OCTANE}$

FROM (iv)

$= 0.7995 \frac{mol}{km} \times -5530\ kJ/mol$

$= -4421\ kJ/km$

THE PROBLEM NOW SIMPLIFIES TO, "HOW MUCH DOES IT COST TO BUY LPG CONTAINING −4421 kJ OF ENERGY?", AND WE CAN RE-APPLY MOST OF THE PREV. STEPS FOR GASOLINE.

$n_{PROPANE\ NEEDED} = \frac{E}{\Delta H^{\ominus}_{c,\ PROP}} = \frac{-4421\ kJ}{-2220\ kJ/mol} = 1.991\ mol$

$m_{PROPANE} = n \cdot M$

$= 1.991\ mol \cdot 44.1\ g/mol = 87.8\ g$

$V_{PROPANE} = \frac{m}{\rho} = \frac{87.8\ g}{0.5853\ g/cm^3} = 150.0\ cm^3$

$= 0.1500\ dm^3$

$\$_{PROPANE} = V \cdot \$_{UNIT\ COST}$

$= 0.1500\ dm^3 \times 0.60\ \$/dm^3 = 0.09 \frac{HK\$}{km}$

(d) Calculate the fuel savings after an engine switch from gasoline to LPG. Assuming that a car would take the same amount of energy to run.

(d) 0.144 HK$/km

FOR EACH km, COST IS LOWERED BY:

0.234
− 0.090
0.144

(e) i. Estimate the distance a taxi travel in a day.

i. 800 km

ASSUME 2× 10 hr SHIFTS = 20 hr.
ESTIMATE AN AVG SPEED OF 40 km/hr,
TOTAL DISTANCE = 40 km/hr × 20 hr = 800 km

ii. Calculate the fuel saving each day.

ii. $115.2

DAILY SAVING = DISTANCE × SAVINGS PER km
= 800 km × 0.144 HK$/km = $115.2

iii. Propose a reasonable subsidy for the engine change, outlining your reason.

iii. $ 0

COST OF ENGINE CHANGE = $16000
DAILY SAVING = 115.2 $/day
DAYS TO RECOUP COST = 16000 $ / 115.2 $/day ≃ 140 days.
THE SWITCH IS ECONOMICALLY SOUND ON ITS OWN AFTER 4 MONTHS. SHORT ENOUGH TO WARRANT NO SUBSIDY.

5 Energy Density and Specific Energy

4. **Energy density**

Energy density describes the energy contained in a fixed volume. A higher energy density source is more compact and more convenient to carry.

Energy density is calculated as $\frac{E}{V}$, where E is the amount of energy released and V is the volume occupied by the fuel. It is often given as the SI unit $\frac{\text{kJ}}{\text{m}^3}$

5. (a) State the amount of energy released by combusting 1.00 dm^3 of gasoline (octane).

(a) $-3.401 \times 10^4 \ \frac{\text{kJ}}{\text{dm}^3}$

(b) Convert the value into the SI unit of $\frac{\text{kJ}}{\text{m}^3}$

(b) $-3.401 \times 10^7 \ \text{kJ/m}^3$

$1 \text{ m}^3 = 1\text{m} \times 1\text{m} \times 1\text{m}$

$= 10 \text{ dm} \times 10\text{dm} \times 10\text{dm} = 1000 \text{ dm}^3$

$-3.401 \times 10^4 \frac{\text{kJ}}{\text{dm}^3} \times \frac{1000 \text{ dm}^3}{1 \text{ m}^3} = -3.401 \times 10^{4+3} \frac{\text{kJ}}{\text{m}^3}$

6. Explain, showing your calculations, whether gasoline or LPG has a higher energy density.

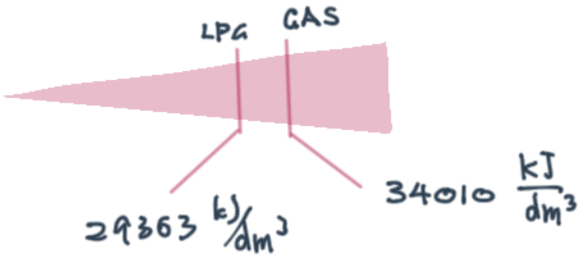
6. GASOLINE.

$m_{1\,\text{dm}^3\,\text{LPG}} = V \times \rho$

$= 1.00 \text{ dm}^3 \times 0.5853 \text{ kg/dm}^3 = 585.3 \text{ g}$

$n_{1\,\text{dm}^3\,\text{LPG}} = \frac{m}{M_{\text{propane}}} = \frac{583.3 \text{ g}}{44.1 \text{ g/mol}} = 13.2 \text{ mol}$

$q_{1\,\text{dm}^3\,\text{LPG}} = n \cdot \Delta H^{\ominus}_{c\,C_3H_8} = -29363 \text{ kJ}$



7. **Specific energy**

Specific energy describes the energy released from a fixed mass. A source with higher specific energy is lighter.

Specific energy is calculated as $\frac{E}{m}$, where E is the amount of energy released and m is the volume occupied by the fuel. It is often given as the SI unit $\frac{\text{kJ}}{\text{kg}}$

(a) State the amount of energy released by combusting 1.00 dm^3 of gasoline (octane).

(a) $-3.401 \times 10^4 \frac{kJ}{dm^3}$

(b) Calculate the specific density of gasoline.

(b) -48410 kJ/kg

$m_{1 dm^3 \text{ gasoline}} = V \cdot \rho = 0.7025 \text{ kg}$

SPECIFIC DENSITY $= E/m = \frac{-34010 \text{ kJ}}{0.7025 \text{ kg}} = -48412$ kJ/kg

(c) Explain, showing your calculations, whether gasoline or LPG has a higher specific energy.

(c) LPG

$\text{SPEC. } E_{LPG} = \frac{E}{m} = \frac{-29363 \text{ kJ}}{0.585 \text{ kg}} = -50193$ kJ/kg

NOTICE THAT DUE TO SUBSTANCES HAVING DIFFERENT DENSITIES, THEY COULD RANK FAVORABLY WRT ENERGY DENSITY BUT UNFAVORABLY WRT SPECIFIC ENERGY.